Applying Stochastic Attributed C-Set Rewriting in Agent-based Modeling in Public Health & Beyond

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[7]University of Southhampton

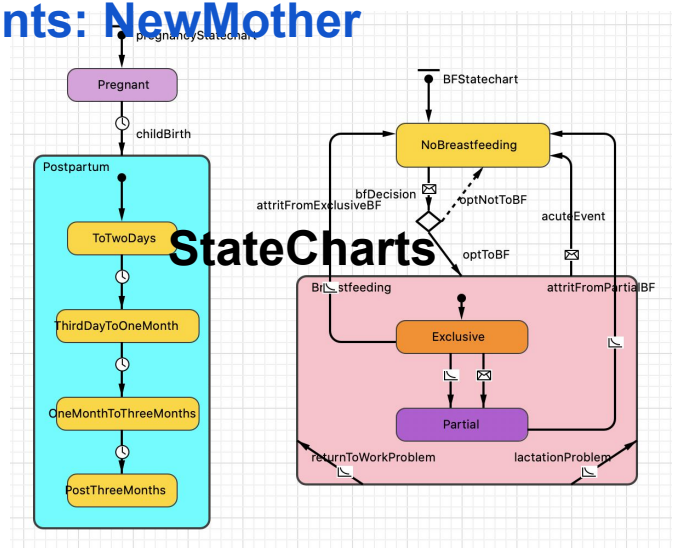
Introduction

Agent-Based Modeling (ABM, also named Individual-Based Modeling) is:

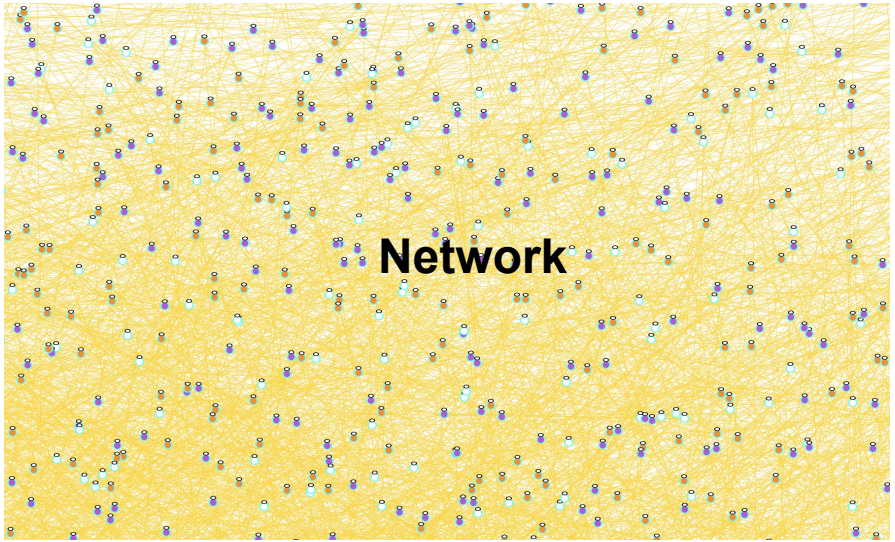
- A computational method to simulate complex systems
- Focuses on emergent behavior from interactions between:
  - Individually characterized agents
  - Their environment
  - Other agents
- Stochastic

A Traditional ABM Model Example

Agents: NewMother



StateCharts



Network

```
int totNumPop = women.size();
double sum_bfIntention = 0;

// before the iteration, assign the current bf intention value to a copy

for (int i = 0; i < totNumPop; i++) {
    women.get(i).bfIntentionNotUpdatedRightNow = women.get(i).bfIntention;
}

// then, the bfIntention is calculated based on the neighbours original bf intention, instead of

for (int i = 0; i < totNumPop; i++) {
    //women.get(i).bfIntention = 0.5 * women.get(i).bfIntention + 0.5 * women.get(i).meanBFInten
    //modified 2019.11.03
    double raw_bfIntention = 0.5 * women.get(i).bfIntention + 0.5 * women.get(i).meanBFIntention
    women.get(i).bfIntention = randomTrue(raw_bfIntention)? 1.0:0.0;
    diff_bfIntention.add(women.get(i).getIndex(), women.get(i).bfIntention - women.get(i).meanBF
    }

for (int i = 0; i < totNumPop; i++) {
    sum_bfIntention += women.get(i).bfIntention;
    }

double average_bfIntention = sum_bfIntention/totNumPop;

return average_bfIntention;
//traceln("average_bfIntention: " + average_bfIntention);
```

Logic in Code

```
int totNumPop = women.size();
double sum_bfInitiation = 0;

// before the iteration, assign the current bf intention value to a copy

double bfint;
double bfhostatus;
double exb;

for (int i = 0; i < totNumPop; i++) {
    bfint = women.get(i).bfIntention;
    bfhostatus = women.get(i).babyFriendlyHospital;
    exb = exp(0.7284+2.5034*bfint + this.causalEffectOfDeliveryAtBabyFriendl
    //women.get(i).bfInitiation = ;
    double raw_bfInitiation = exb / (1+exb);
    women.get(i).bfInitiation = randomTrue(raw_bfInitiation)? 1.0 : 0.0;
}

for (int i = 0; i < totNumPop; i++) {
    sum_bfInitiation += women.get(i).bfInitiation;
    }

double average_bfInitiation = sum_bfInitiation/totNumPop;

return average_bfInitiation;
//traceln("average_bfInitiation: " + average_bfInitiation);
```

Intervention Parameters

- knowledgeScore_BetaDist_a
- knowledgeScore_BetaDist_b
- rateOfDeliveryAtBFFH
- rateOfProfSupp
- rateOfFamSupp
- rateOfWorkPlaceSupp

Motivation

Traditional ABM Challenges:

- Expressing complex structures
  - Limited by object-oriented programming techniques.
- Obscured model logic
  - Extensive software engineering hides core insights.
- Interdisciplinary barriers
  - Difficult for diverse teams to critique and improve models.
- Lack modular representation and compositional construction

A Stochastic Attributed C-Set Rewriting Framework

A Category C (with attributes)

A collection of rewrite rules

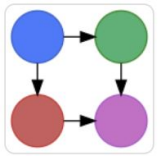
For each rewrite rule p, pair with a timer, which is either

[State dependent] For each timestep, perform Bernoulli draw based on hazard rate

[State-independent] Pair with a random sampled time t (the fired time of this rewrite rule)

[State-independent, Memoryless process] Schedule only first of events with hazard rate (# matches)λ

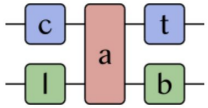
Software Implementation



AlgebraicJulia

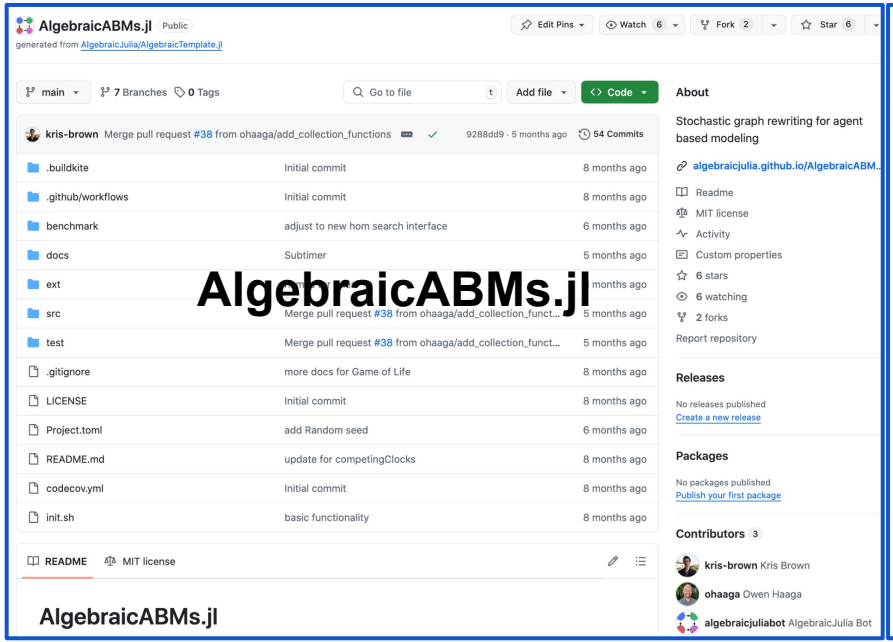
An Ecosystem of Software Based on Generalized Algebra and Category Theory in Julia

169 followers https://www.algebraicjulia.org

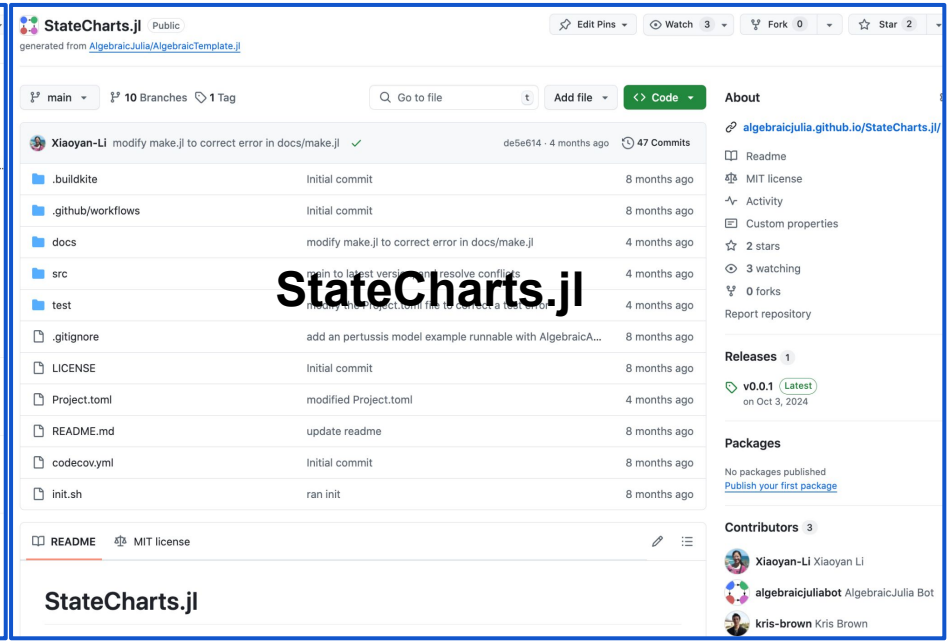


Catlab.jl

Catlab.jl is a framework for applied and computational category theory



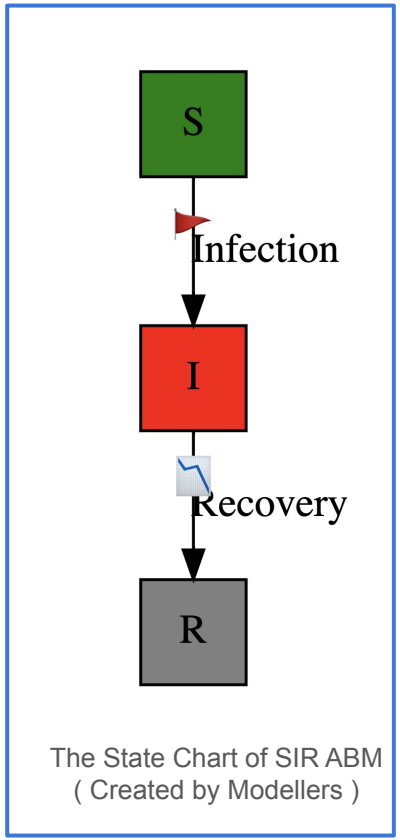
AlgebraicABMs.jl



StateCharts.jl

An Example: An SIR (Susceptible-Infectious-Recovered) Infectious Disease ABM model

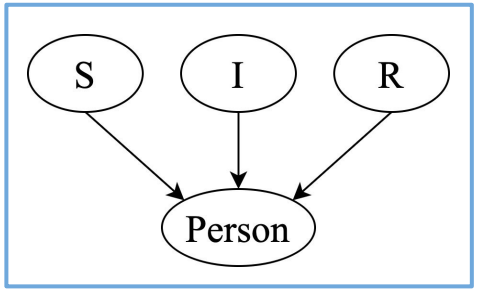
1. Define State Charts (Example)



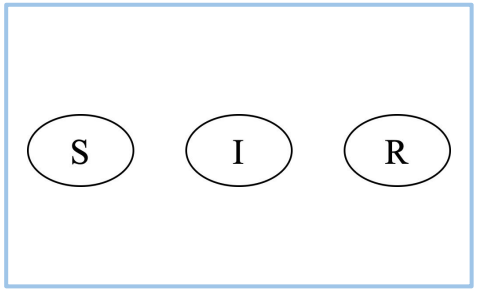
The State Chart of SIR ABM
(Created by Modellers)

Automatically generate the ABM model schema (Category C)

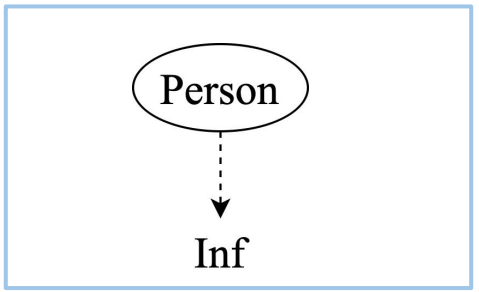
Category C



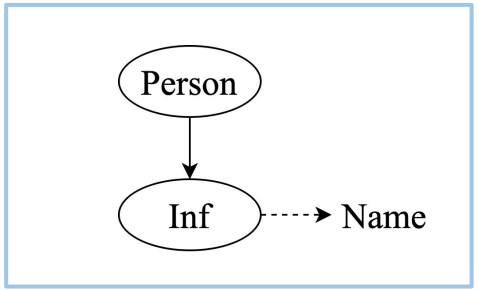
Option 1



Option 2



Option 3



Option 4

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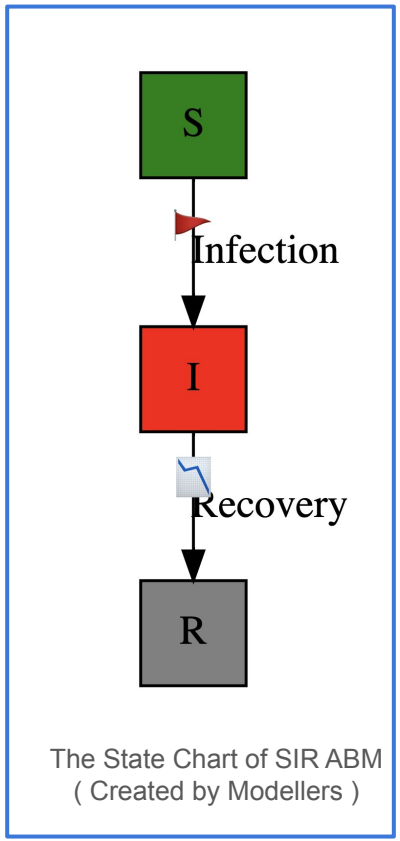
1. Define State Charts (Example)



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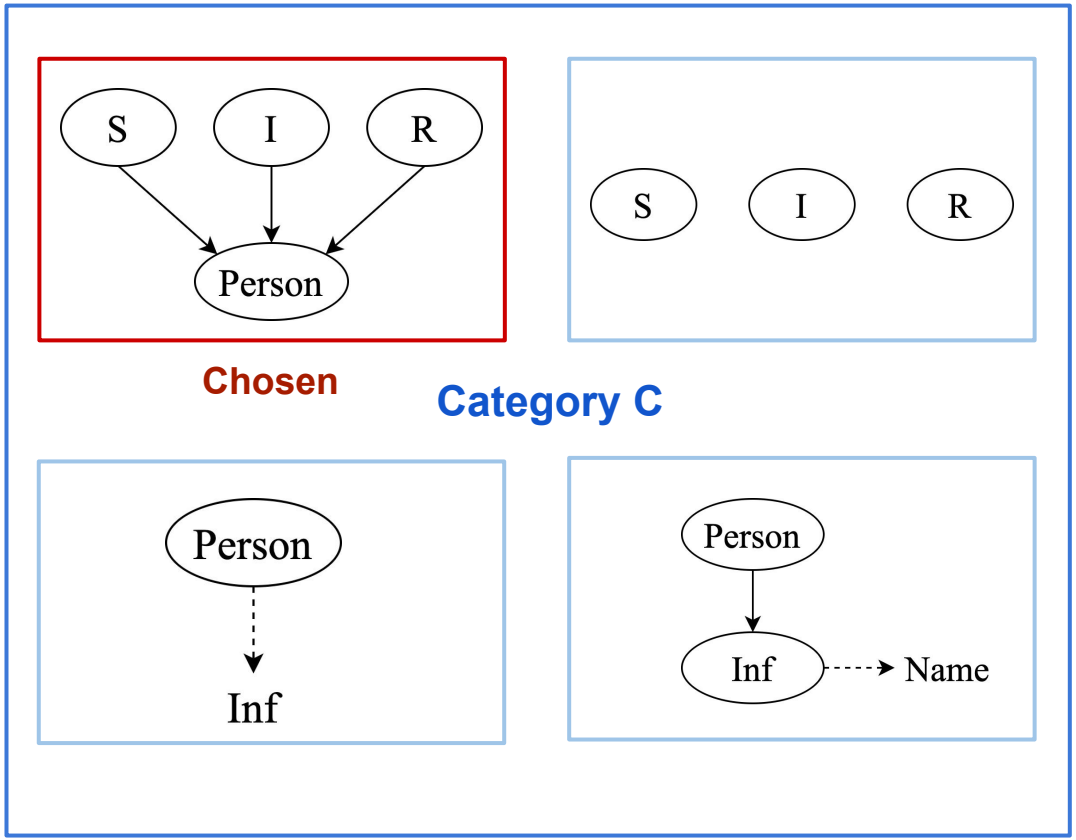
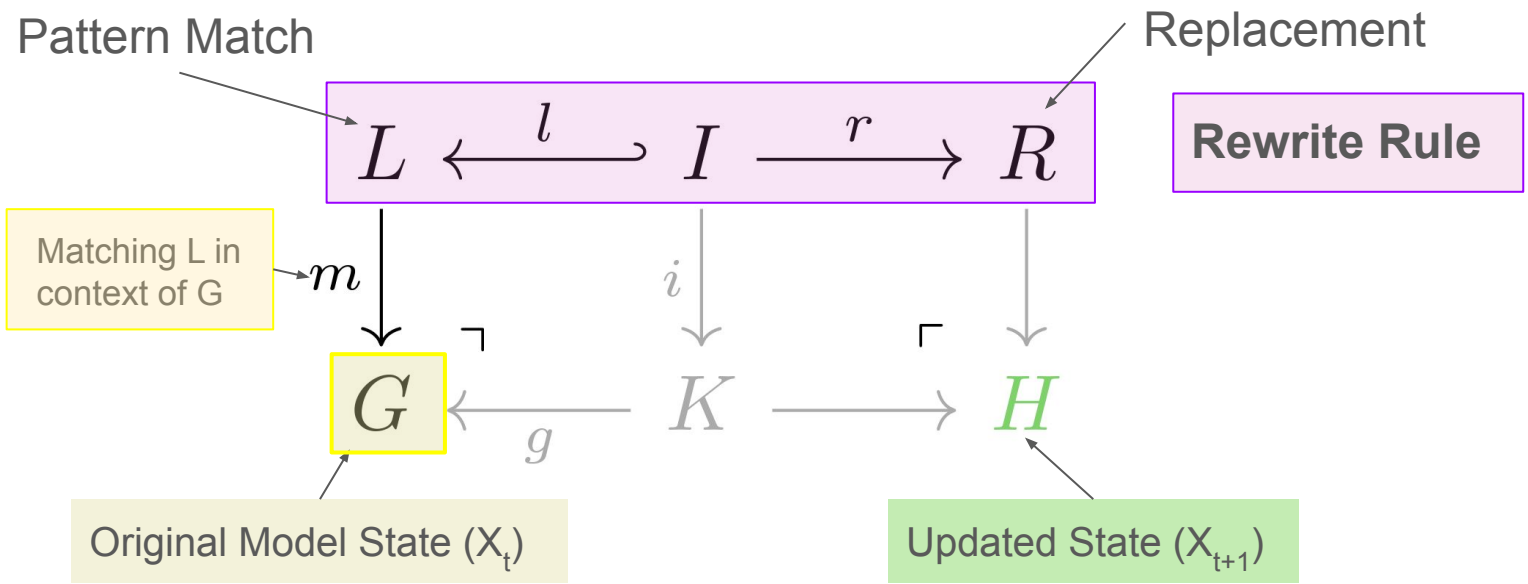
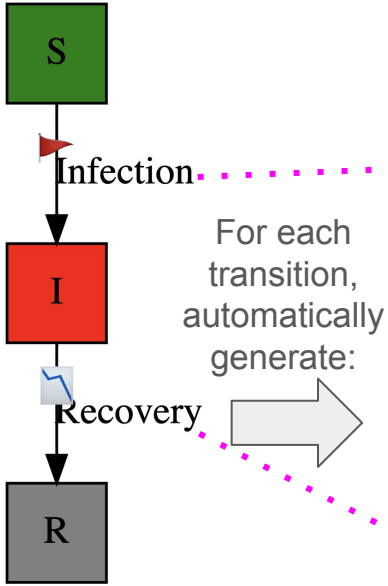


Graphical Rewriting: Double Pushout (DPO)

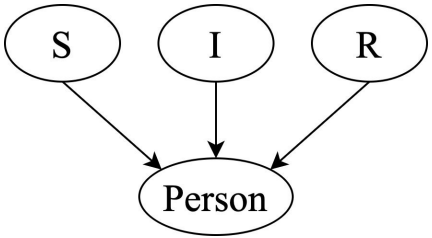


L, I, R, G, K, H are all objects in the category of $\mathrm{Set}^{\mathrm{C}}$.

An Example: An SIR (Susceptible-Infectious-Recovered) Infectious Disease ABM model

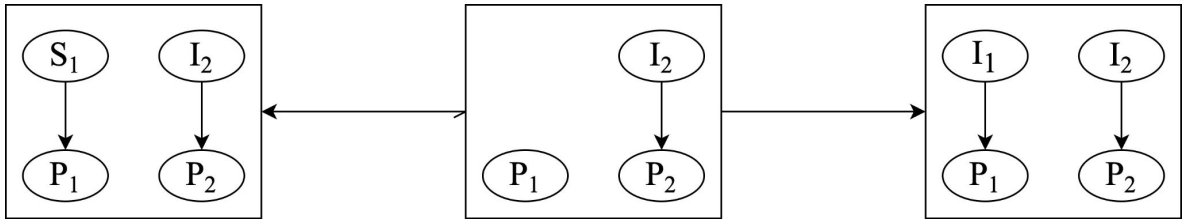


For each transition, automatically generate:



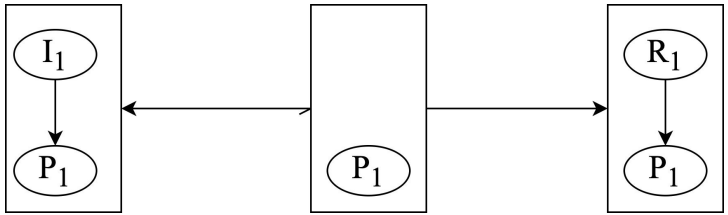
Pair of (Rewrite Rules, Timer):

- Transition “Infection”:
  - Rewrite Rule:



  - Timer: Exponential(ContactRate * Infectivity)

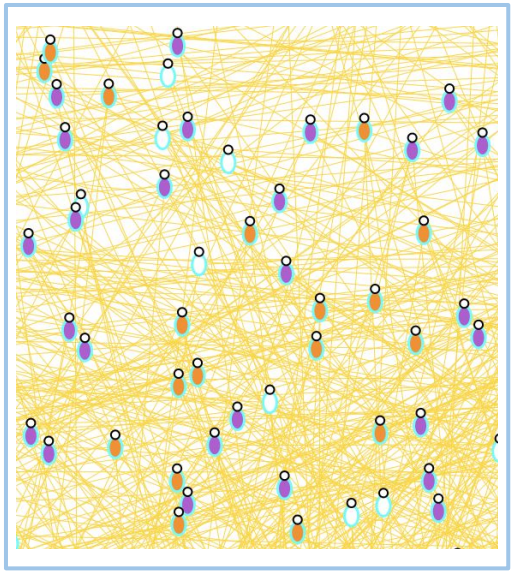
- Transition “Recovery”:
  - Rewrite Rule:



  - Timer: Exponential(1 / average_illness_duration)

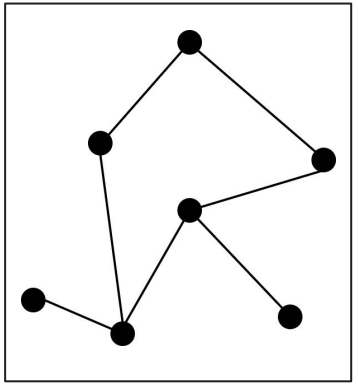
An Example: An SIR (Susceptible-Infectious-Recovered) Infectious Disease ABM model

2. Define Networks



An example of networks representing **connections** between agents

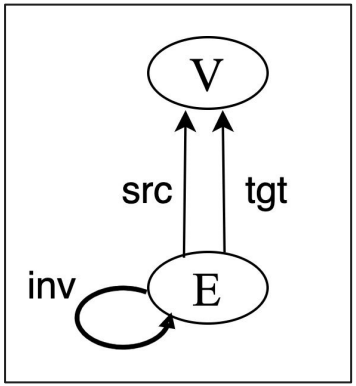
Abstracted to



Symmetric Graphs

Category C

(Schema)



Category C

An Example: An SIR (Susceptible-Infectious-Recovered) Infectious Disease ABM model

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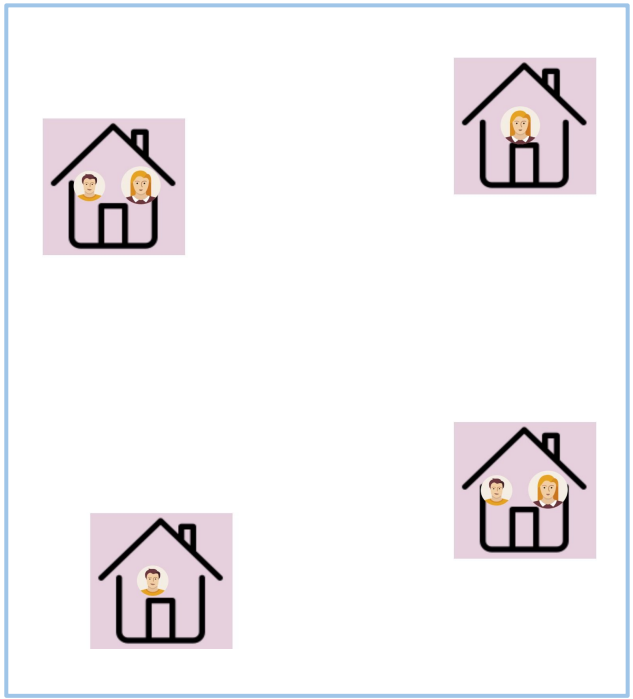
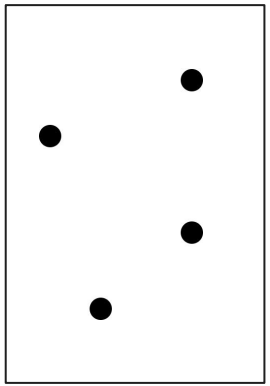


An example of networks representing **disconnected** homes

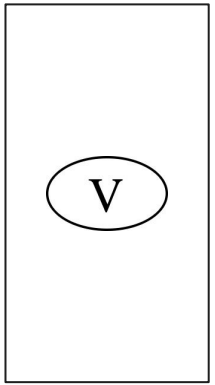
Abstracted to



Discrete Graphs

Category C

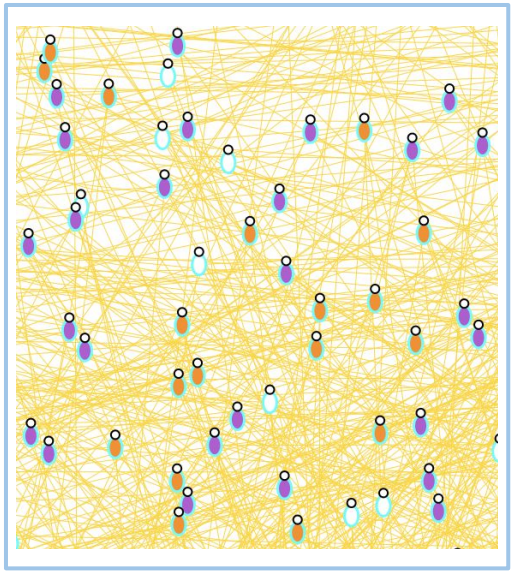
(Schema)



Category C

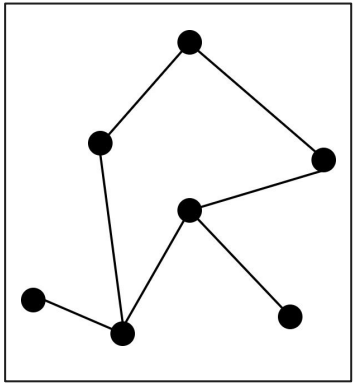
An Example: An SIR (Susceptible-Infectious-Recovered) Infectious Disease ABM model

2. Define Networks



An example of networks representing **connections** between agents

Abstracted to

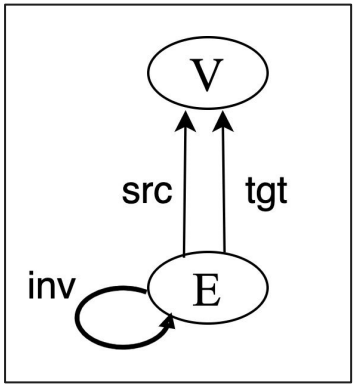


Symmetric Graphs

Category C

(Schema)

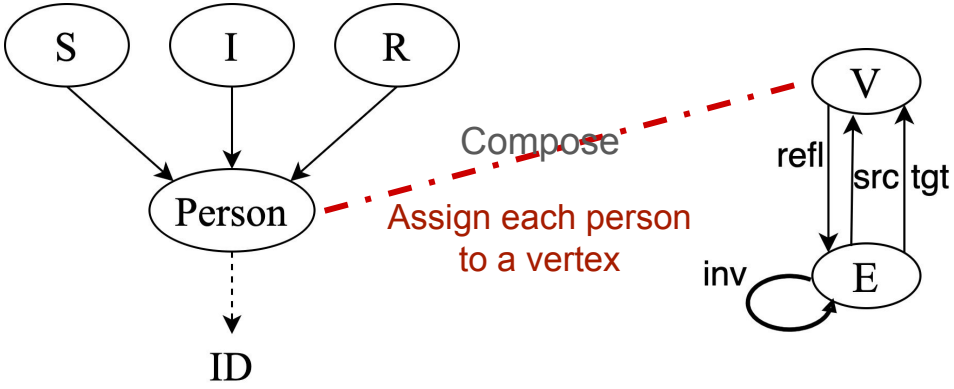
Used



Category C

An Example: An SIR (Susceptible-Infectious-Recovered) Infectious Disease ABM model

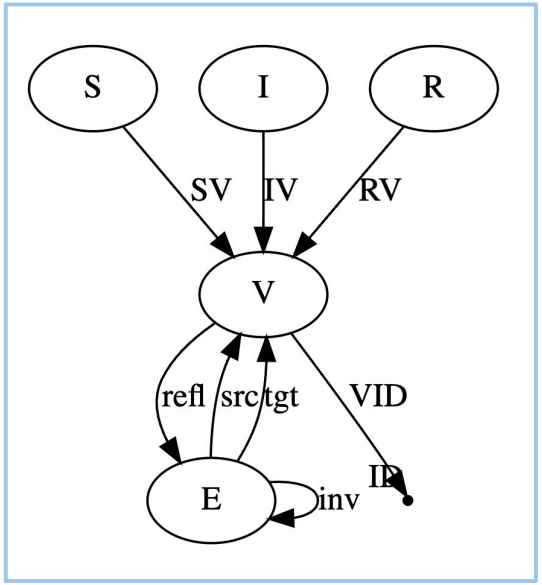
3. ABM Model's Category C′ (Schema): Composing Categories of Multi-parts



Schema Category only Represents States of Agents

Schema Category of the Network

The model's Category:



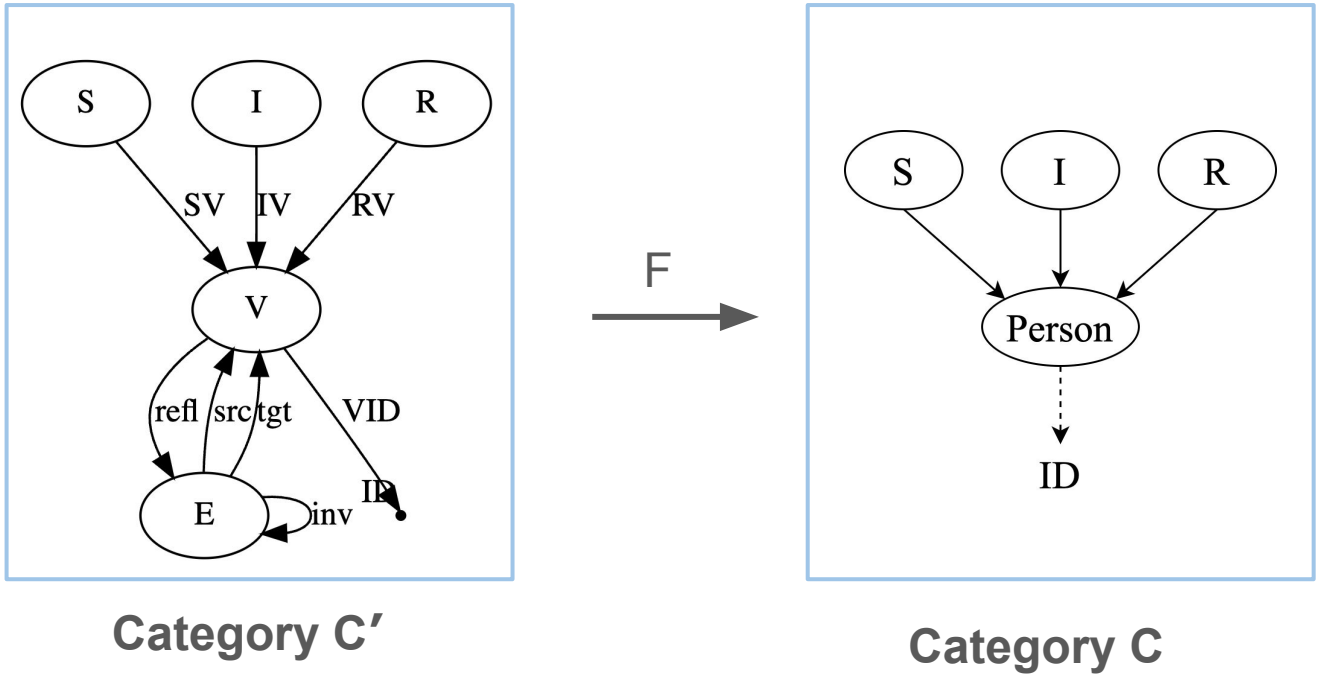
The Final Model's Category C′

Composition via pushout

An Example: An SIR (Susceptible-Infectious-Recovered) Infectious Disease ABM model

3. Generate the Rewrite Rules of Final Model Category Using Functorial Data Migration

We can get a functor **G: C-Set → C′-Set** by the functor **F: C′ → C**

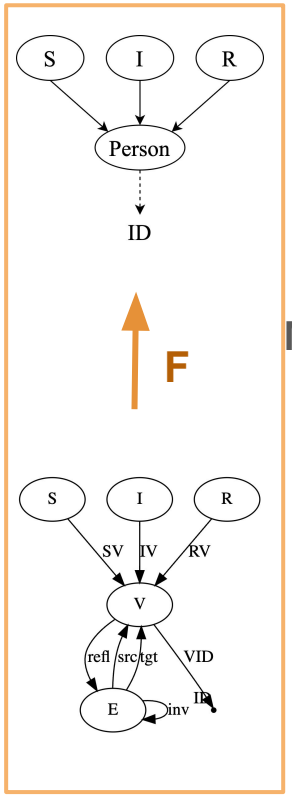


Category C′ **Category C**

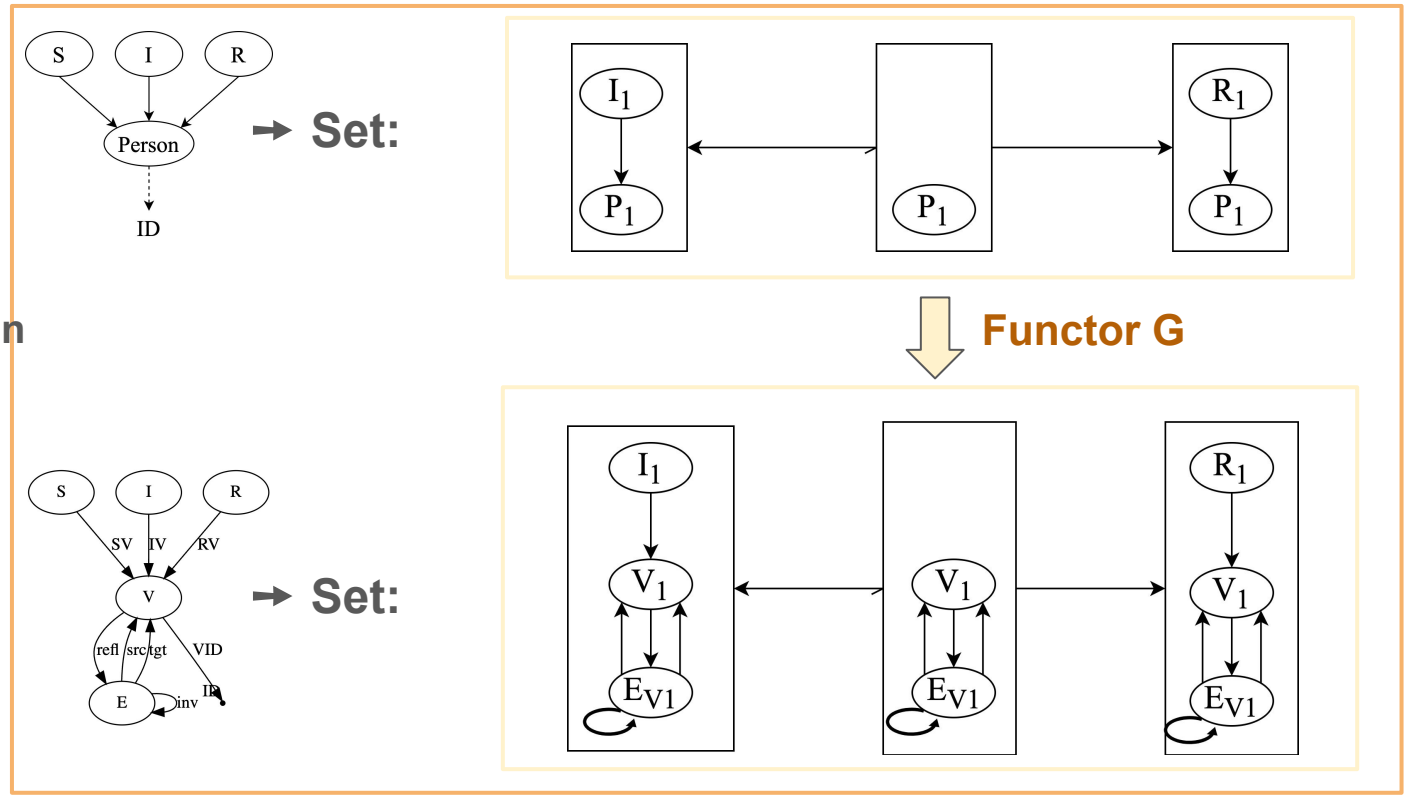
An Example: An SIR (Susceptible-Infectious-Recovered) Infectious Disease ABM model

Transition “Recovery”

3. Generate the Rewrite Rules of Final Model Category Using Functorial Data Migration



Data Migration

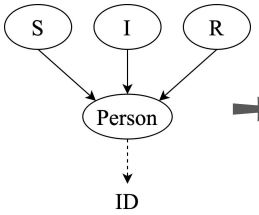


Note: For better visualization, the attributes of id are not plotted out

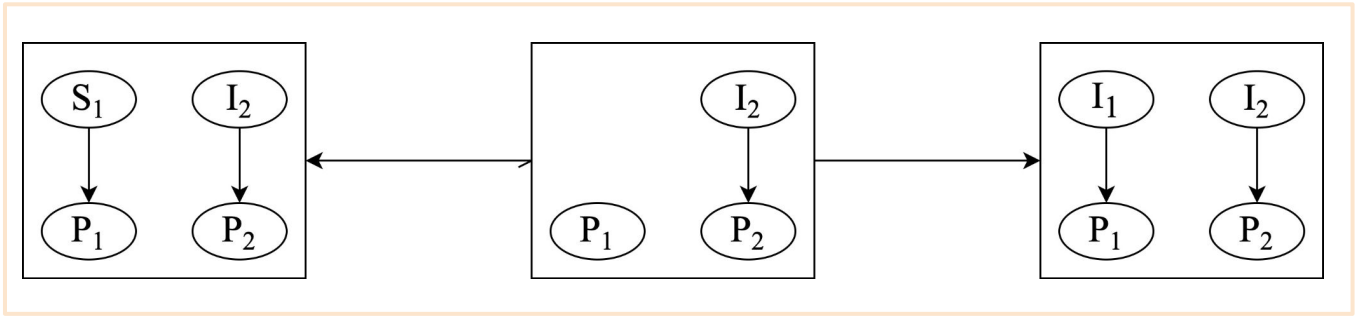
An Example: An SIR (Susceptible-Infectious-Recovered) Infectious Disease ABM model

Transition "Infection"

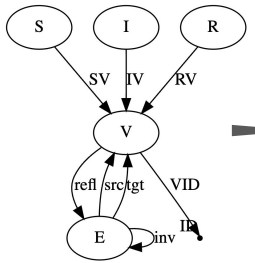
3. Generate the Rewrite Rules of Final Model Category Using Functorial Data Migration



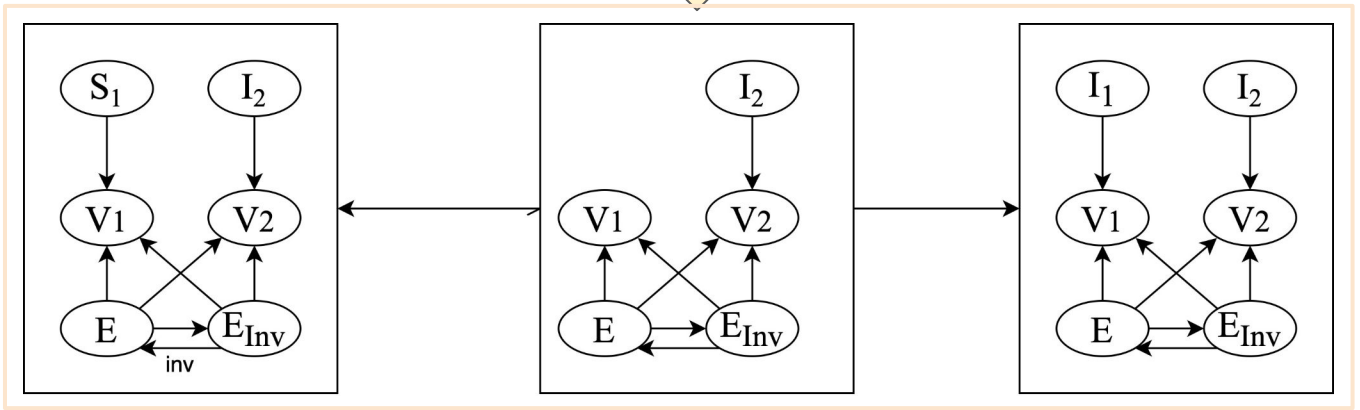
→ **Set:**



Functor G



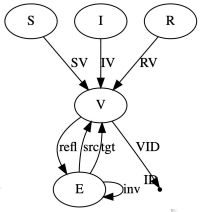
→ **Set:**

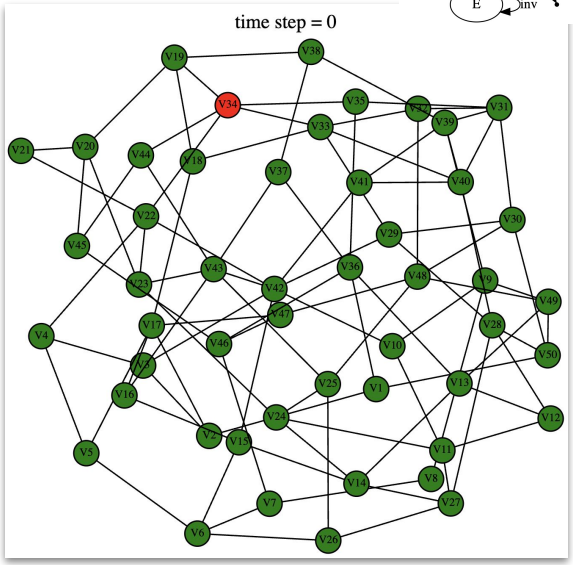


Note: For better visualization, the reflective edges and attributes of id are not plotted out

An Example: An SIR (Susceptible-Infectious-Recovered) Infectious Disease ABM model

4. Initialization





Small World Network

Categorical database

S→49, I→1, R→0, V→50, E→244

S	SV
1	1
2	2
3	3
…	…

I	IV
1	34

R	RV

SV IV RV

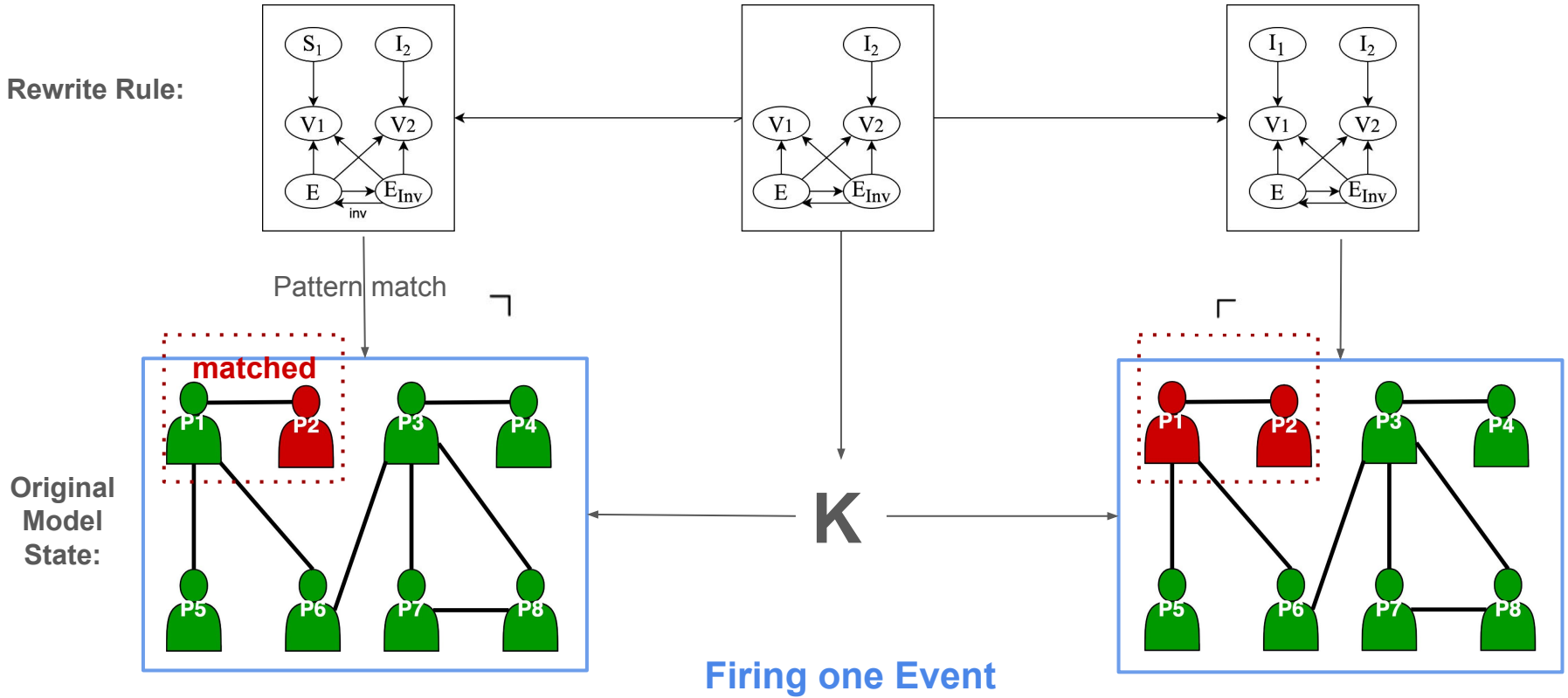
V	refl	VID
1	1	“V1”
2	2	“V2”
3	3	“V3”
…	…	…

inv src tgt refl

E	src	tgt	inv
1	1	1	1
2	2	2	2
3	3	3	3
…	…	…	…

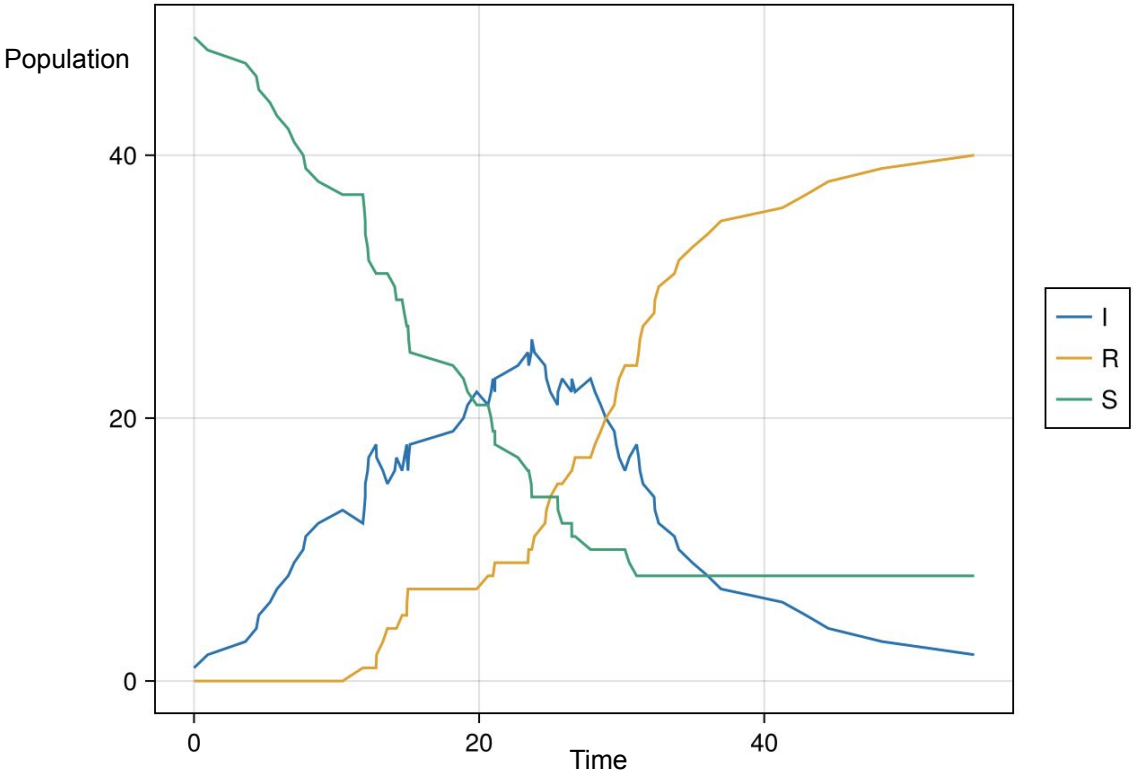
An Example: An SIR (Susceptible-Infectious-Recovered) Infectious Disease ABM model

4. Computation of ABM using Categorical Rewriting (DPO) to Fire Events



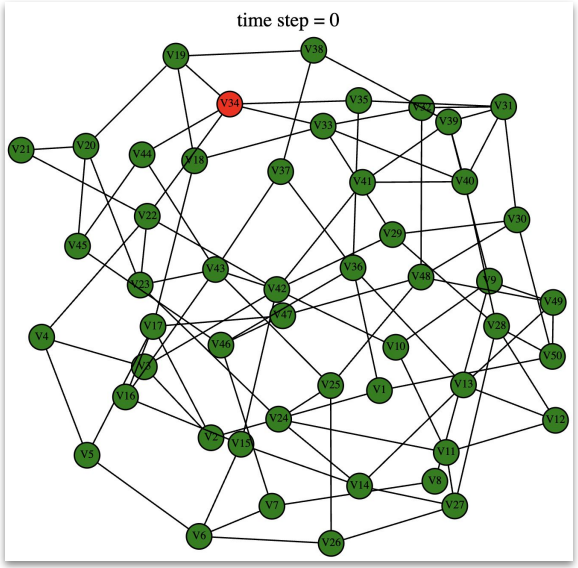
An Example: An SIR (Susceptible-Infectious-Recovered) Infectious Disease ABM model

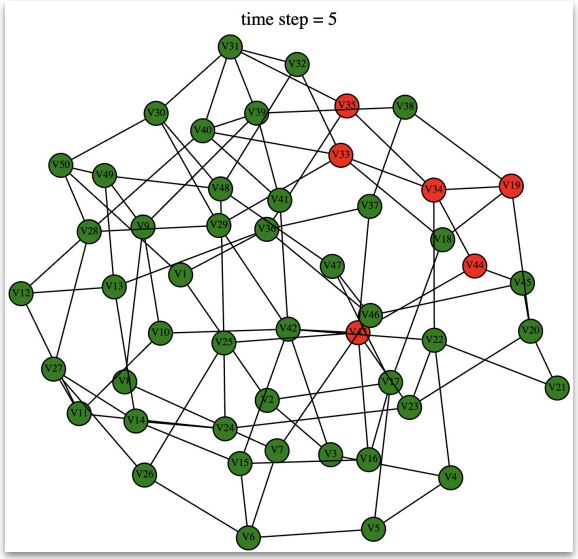
5. Results

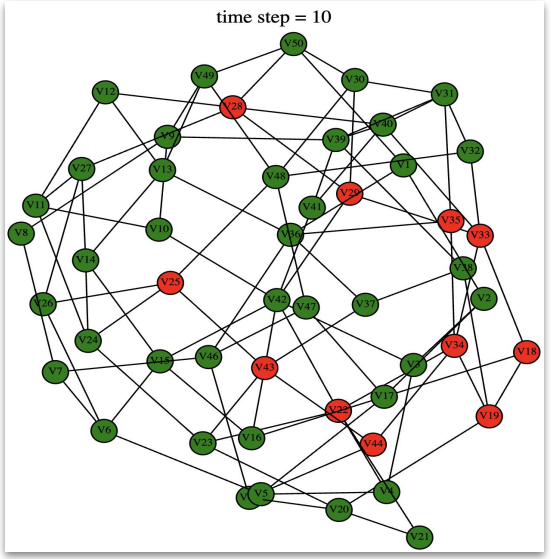


An Example: An SIR (Susceptible-Infectious-Recovered) Infectious Disease ABM model

5. Results

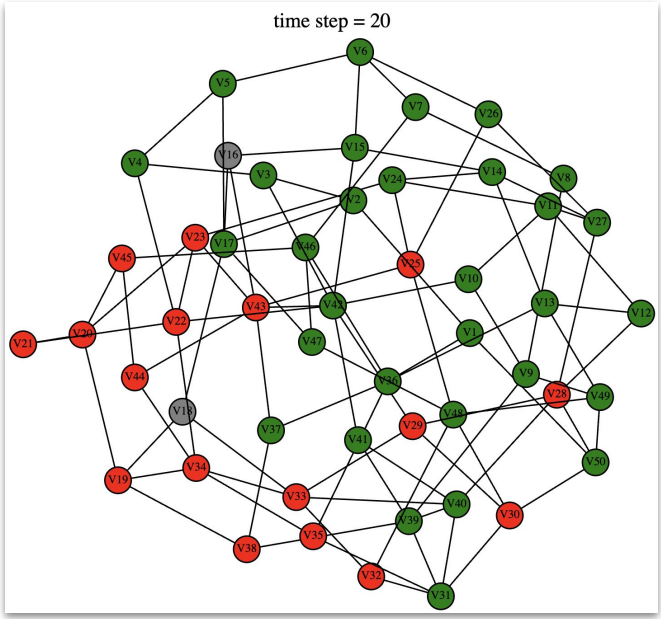


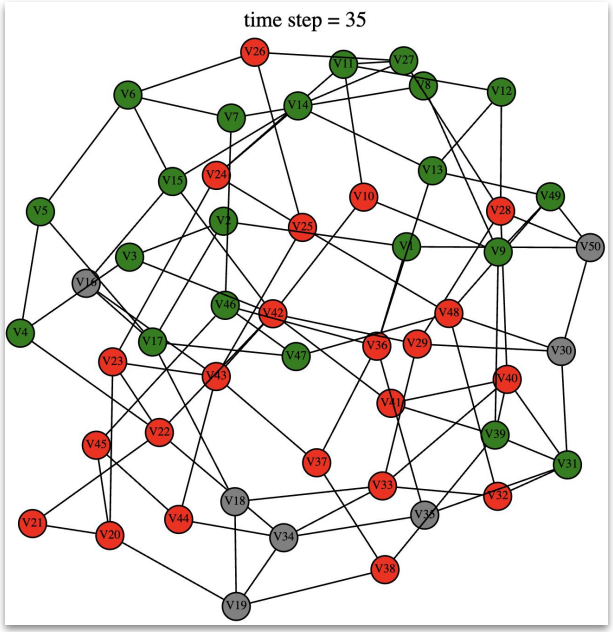


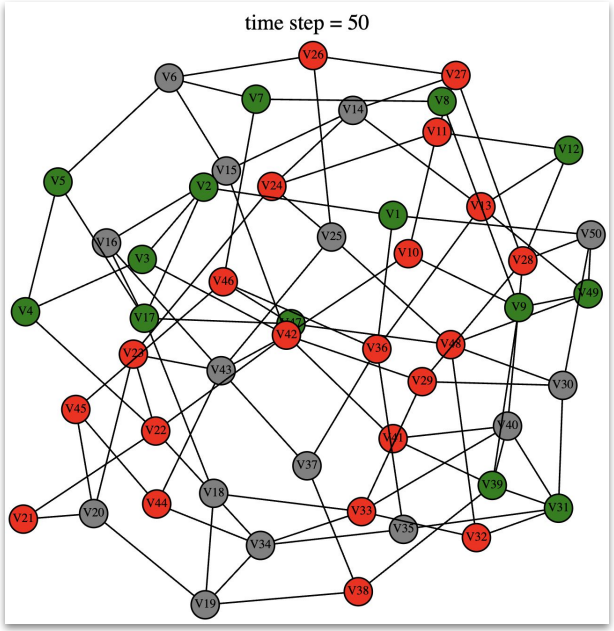


An Example: An SIR (Susceptible-Infectious-Recovered) Infectious Disease ABM model

5. Results







Take-Home Messages

- Stakeholder transparency
- Contributes a mathematical framework for representing ABM structures and computations using ACT
- Avoiding getting bogged down in unwieldy & large codebase
- Representing ABM models modularly
- Composability: Capacity to compose models, laterally, hierarchically, etc.
- Capacity to map to other diagram types
- Support for provably safe migration of representation with schema evolution

Q&A

Thank you!