Applications of *Category Theory* to **Advanced Air Mobility Architecture**

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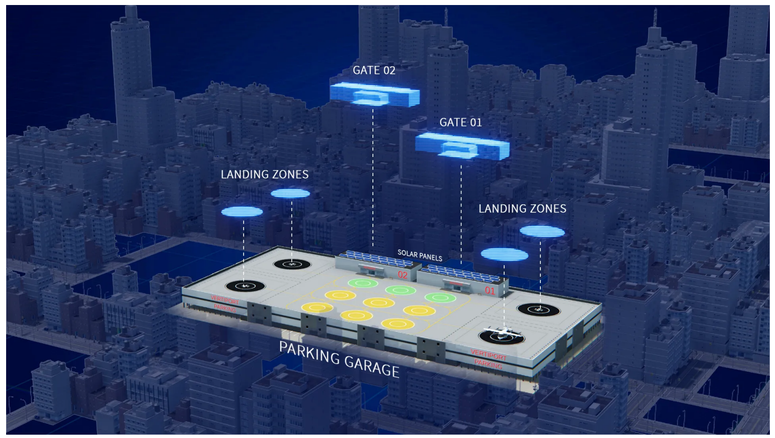
Advanced Air Mobility (AAM)

What **infrastructure** is needed to support **mass air transit**?

Advanced Air Mobility (AAM)

Future **systems** supporting **air transport** that is:

- **Manned** or **autonomous**
- Carrying **passengers** or **cargo**
- **Public** or **private**
- **Intra-city**, **inter-city**, or **regional**
- **Under development**: requires **new technology**, **testing**, and **regulation**

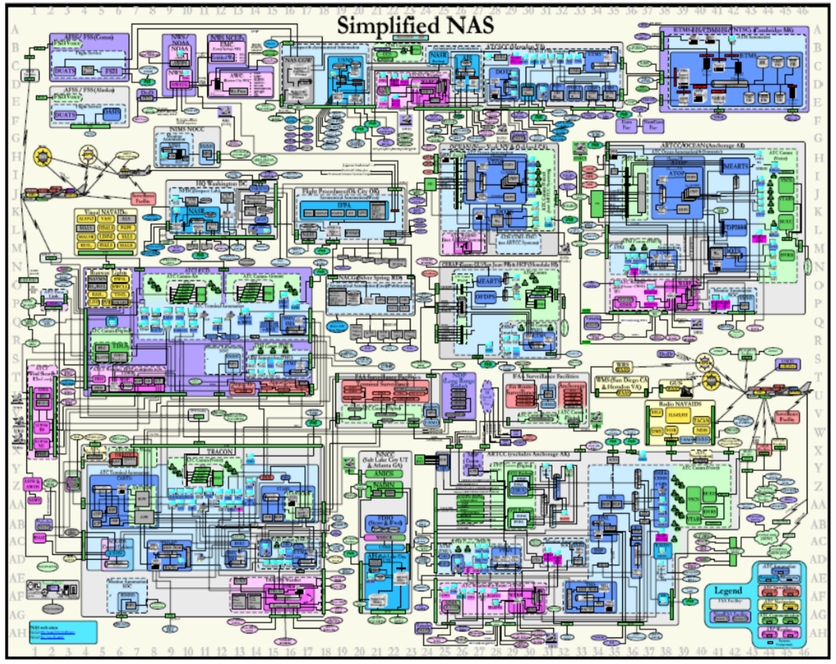


The need for **abstraction**

- **AAM** needs to integrate:
  - new **vehicle types**
  - new **modes of cooperation**
  - **autonomy**

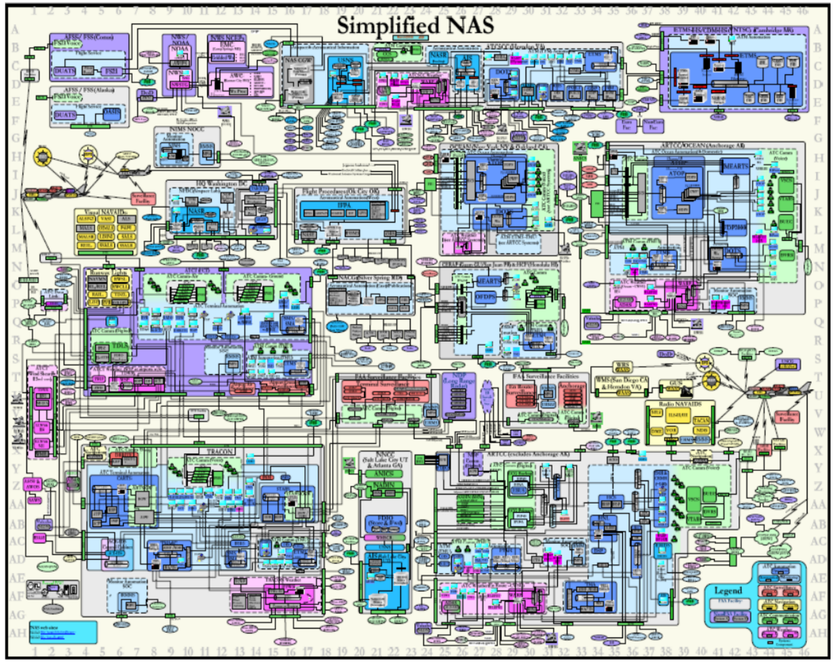
into the **National Airspace System (NAS)**

- **Problem.** How do we **integrate radically new** and **rapidly evolving** functionality into an already **highly complex system** like the NAS?



The need for **abstraction**

- Transportation solutions at risk of developing in **siloes**: need **unifying architecture**
  - **architecture** — shared model depicting structure & behavior of system
  - from **Model-Based System Engineering**
- **Adaptive**, **resilient** architecture essential for **future evolution** of the **NAS**
- **Synthesizing** siloes to identify **unifying patterns** requires **abstraction**



Advantages of **abstraction** via **category theory**

Category theory lets us:

- Draw analogies and **generalize**
  - Build **connections** to foster **mutual comprehension** & **minimize redundancy**
  - Maintain **continuity of methods**
    - e.g. there's a ***category of categories***
- Abstract away details and **formalize**
  - Ensure **precision of language** via **mathematical rigor**
  - Zoom in & out of **different levels of abstraction**
- Leverage the **natural** over the *ad hoc*

Examples of category **theory** & **applications**

From **Fong & Spivak**, *Seven Sketches in Compositionality*, 2018:

- Chapter 2: **Process modularity** with **wiring diagrams** and **monoidal categories**
- Chapter 3: **Data management** with **functors**, **natural transformations**, **adjunctions**, **Kan extensions**, **limits**, and **colimits**
- Chapter 4: **Collaborative design** with **profunctors** and **compact closed categories**

From **Jacobs**, "Objects And Classes, Co-Algebraically," 1996; **Shapiro & Spivak**, *Dynamic Operads, Dynamic Categories*, 2022; **N. & Spivak**, *A Mathematical Theory of Interaction*, 2025:

- **Interaction & evolution** with **polynomial functors** and **coalgebras**

Three stages of system **architecture**

Reference architectures

Specifications of **behaviors** and **requirements**

←

Solution architectures

Proposals to meet specifications in **reference architectures**

←

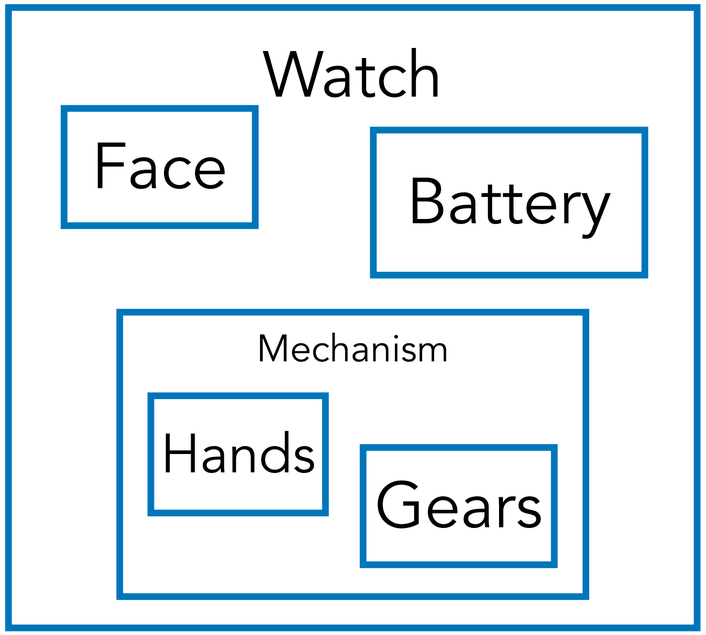
Test architectures

Concrete implementations of **solution architectures**

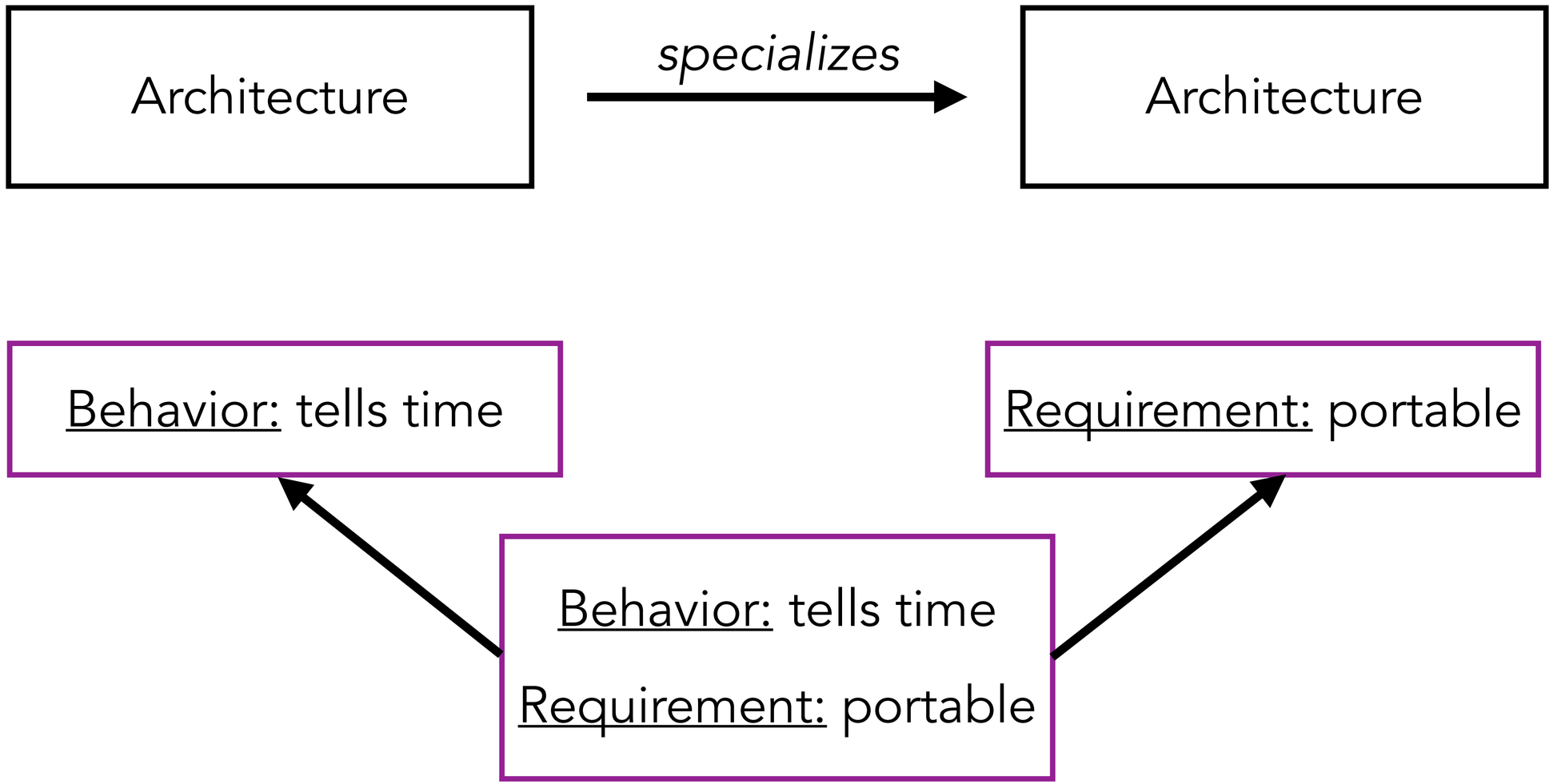
Example

Behavior: tells time

Requirement: portable



Poset (category?) of **architectures**



Poset (category?) of **architectures**

profunctors *indicating* ***compatibility***

Reference architectures

Specifications of **behaviors** and **requirements**

Solution architectures

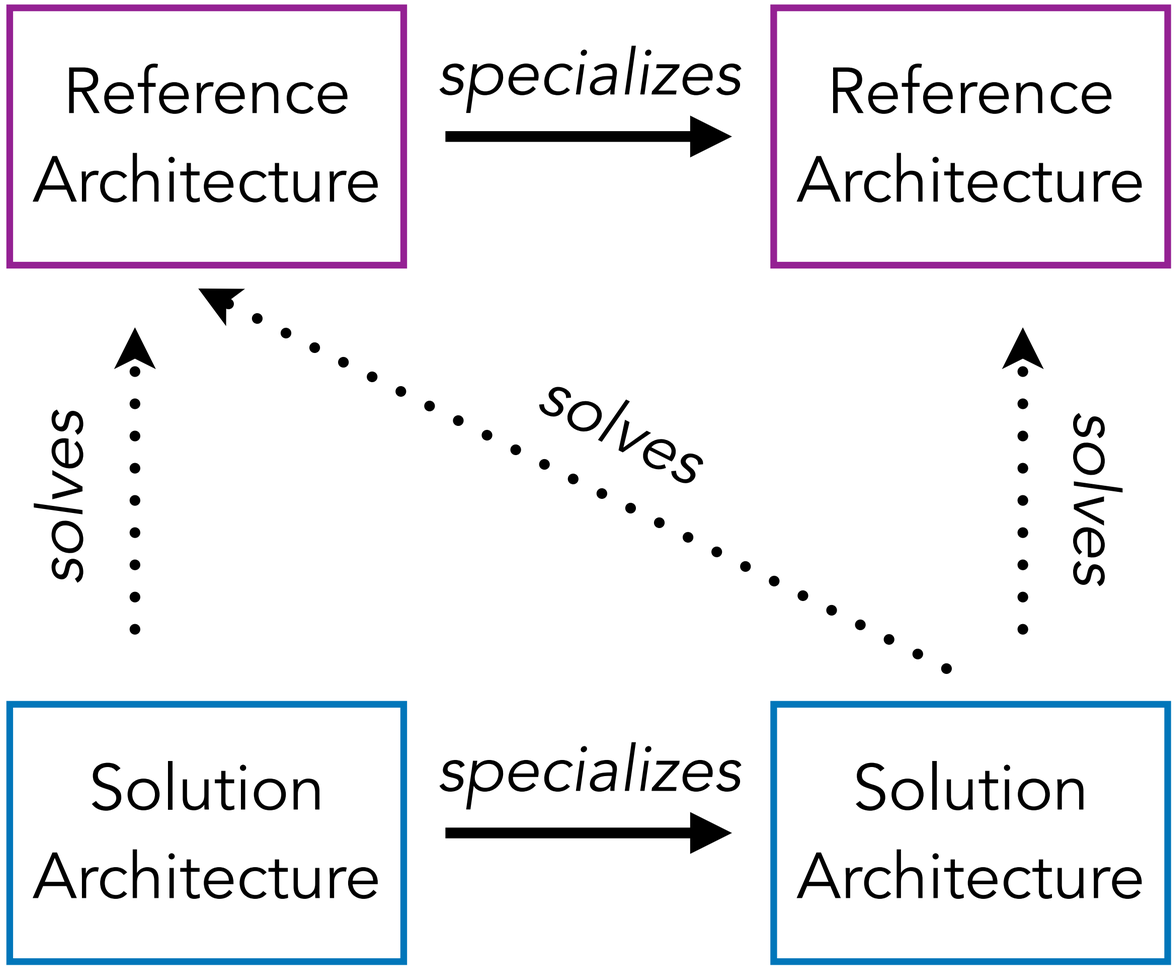
Proposal to meet specifications in **reference architecture**

Test architectures

Concrete implementation of **solution architecture**

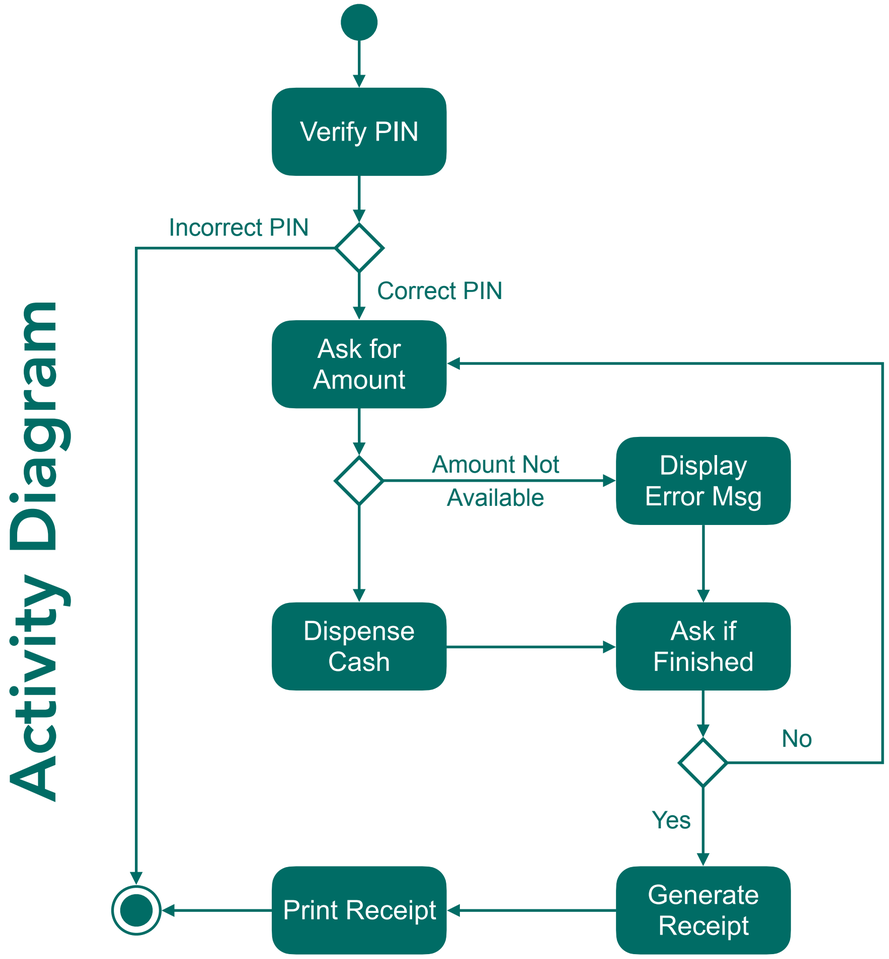
Censi, *A mathematical theory of co-design*, 2015

Poset (category?) of **architectures**

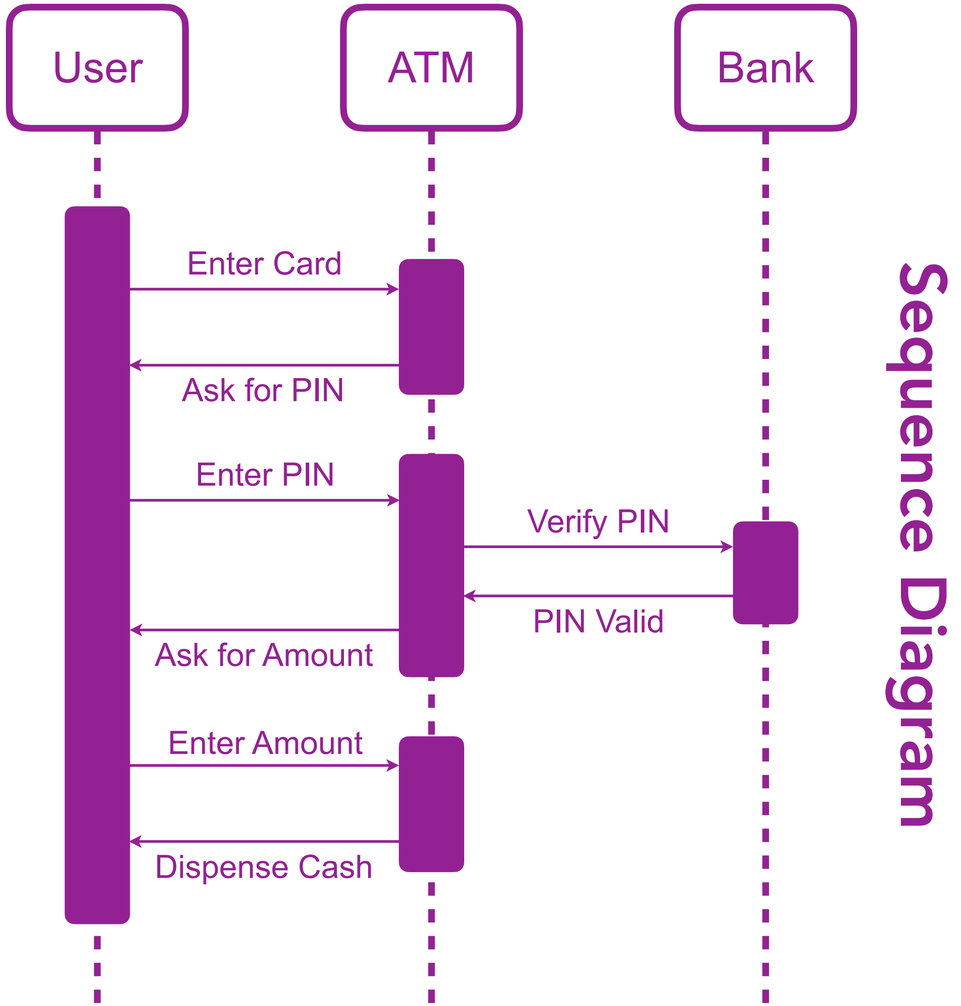


profunctors indicate **compatibility**

Model translation with functorial data migration

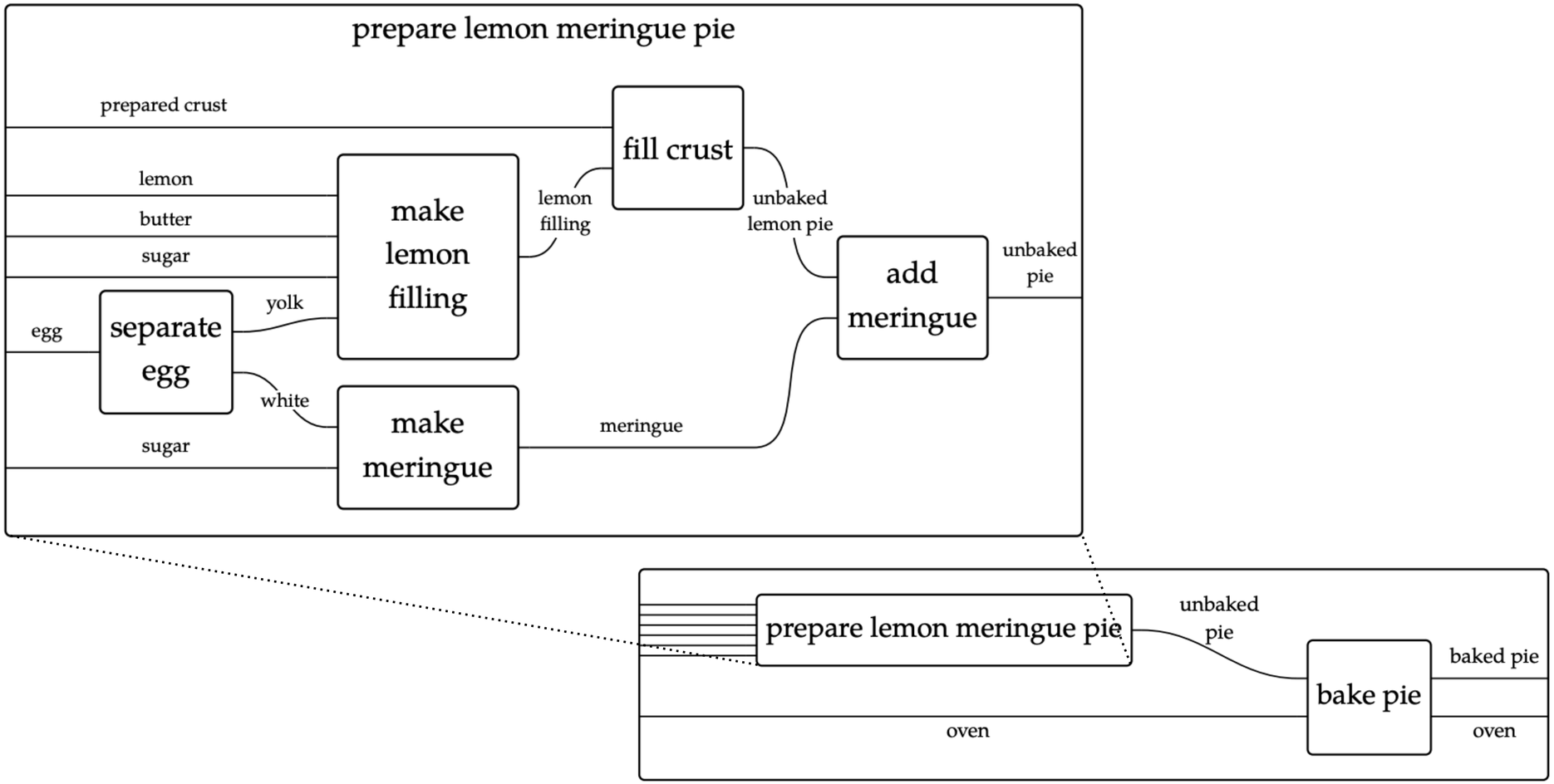


flowchart — **wiring diagram**



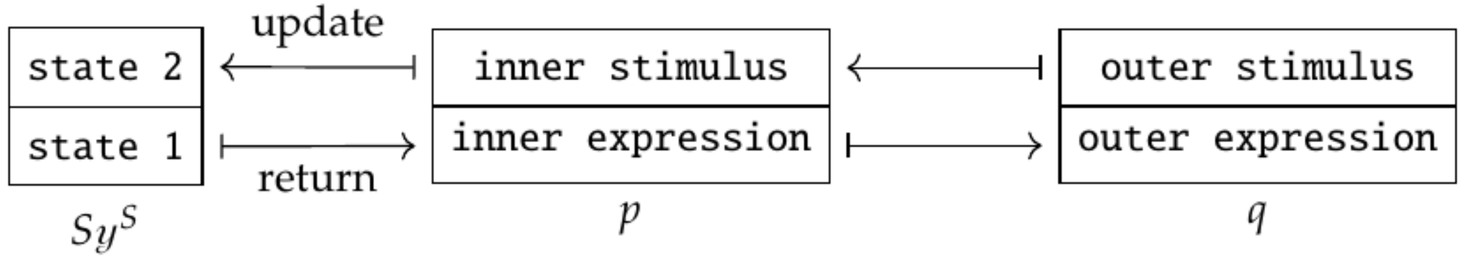
object-oriented interaction — **polynomial functors**

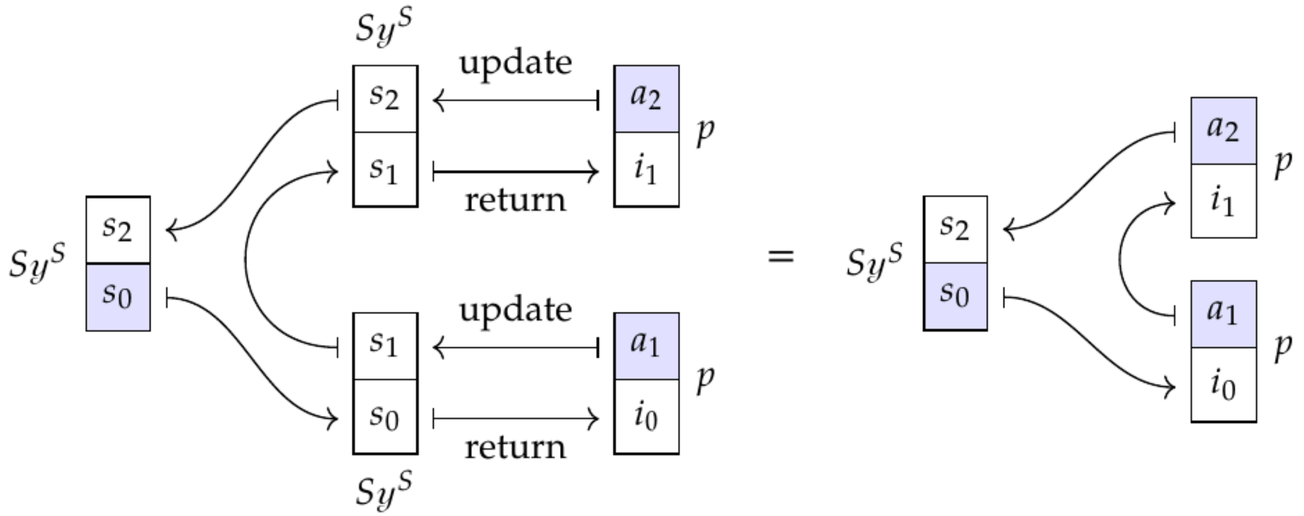
Model **modular, evolving systems & processes** with **monoidal categories, wiring diagrams, & polynomial functors**



Interaction & evolution with polynomial functors

morphisms represent bidirectional **interaction protocols** between **interfaces**





N. & Spivak, *Polynomial Functors: A Mathematical Theory of Interaction*, 2025

Interaction & evolution with polynomial functors

integrate dynamics with polynomial coalgebras and enrichment

Shapiro & Spivak, *Dynamic Operads, Dynamic Categories: From Deep Learning to Prediction Markets*, 2022

Questions?

Want to get involved?

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